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Name of College - S.S. College, J. Bad

Dept - Mathematics

Topic - problem based on System of circles, Radical Axis and Co-Axial Circles.

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Period -

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Important Formulae →

1. $x^2+y^2+2gx+2fy+c=0$ is the - General Equation of circle $(-g,-f)$ is its centre and $\sqrt{g^2+f^2-c}$ is its radius. If circle passes through origin $\Rightarrow c=0$

2. If $r-r_1$ = Distance between the centre $\Rightarrow$ circle Touch each other internally but If $r+r_1$ = Distance between the centre $\Rightarrow$ circle Touch each other Externally.

3. $2gg_1+2ff_1 = c+c_1$ is the required condition under which circles

$$x^2+y^2+2gx+2fy+c=0$$

and $x^2+y^2+2g_1x+2f_1y+c_1=0$

cuts each other orthogonally.

4. Equation of Radical Axis is $2(g-g_1)x+2(f-f_1)y+c-c_1=0$

5. The Radical Axis of two circles is a st. line ⊥ to the line joining the centres.

6. The Radical axis of three circles Taken in pairs are con-current.

1. If a circle cuts two circles orthogonally the Radical Axis of the two circles passes through the centre of the circle.

X 5 Problem based On Circles through the intersection of Circles :→

The Equation of circle passing through the points of intersection of two circles be

$$S_2 + kS_1 = 0$$

the value of k is obtained by Additional Condition or conditions Given in question.

Ex:→ Find the Equation of circles through the point of intersection of the Circles

$x^2+y^2+2x+3y-7=0$ and

$x^2+y^2+3x-2y-1=0$ and passing through the point (1,2)

Solution :→ the Given Circles are

$$x^2+y^2+2x+3y-7=0 \quad \text{---(I)}$$

and $x^2+y^2+3x-2y-1=0$ ---(II)

Let the equation of circle passing through the point of intersection of (I) and (II) be

$$x^2+y^2+3x-2y-1+k(x^2+y^2+2x+3y-7)=0 \quad \text{---(III)}$$

Since it passes through (1,2).

Let the equation of circle passing through the point of intersection of (I) and (II)

$$x^2+y^2-2x-4y+4+k(x+2y-4)=0 \quad \text{---(III)}$$

[using $S+kL=0$]

Since it touches the line $x+2y=0$.

i.e $x=-2y$

then the roots of the equation

$$(-2y)^2+y^2-2(-2y)-4y+4+k(-2y+2y-4)=0$$

put $x=-2y$ in (III)

$$4y^2+y^2+4y-4y+4-4k=0$$

$$\Rightarrow 5y^2+(4-4k)=0$$

Since the roots are equal

there for $D=0$

$$\Rightarrow -4(4-4k)\cdot 5=0$$

$$20-20k=0$$

$$k=1$$

putting $k=1$ in (III)

we have

$$x^2+y^2-2x-4y+4+x+2y-4=0$$

$$\Rightarrow x^2+y^2-x-2y=0$$

this is the required Equation of the circle.

Therefore

$x^2+y^2+3x-2xy-1 + k(x^2+y^2+2x+3y-7)=0$

$\Rightarrow x+y+3-y-x+k(x+y+2+3-7) = 0$

$3 + 6k = 0$

$k = -1/2$

putting $k=-1/2$ in (iii)

$x^2+y^2+3x-2y-1-\frac{1}{2}(x^2+y^2+2x+3y-7)=0$

$2x^2+2y^2+6x-4y-2-x^2-y^2-2x-3y+7=0$

$\Rightarrow x^2+y^2+4x-7y+5=0$

This is the required equation of circle passing through the point of intersection of two circles.

2. Find the equation of the circle passing through the points of intersection of the circles

$x^2+y^2-2x-4y+4=0$

and the st. line

$x+2y-4=0$ and

Touching the line $x+2y=0$

Solution :→ [Equation of circle $S+kL=0$]

value of k is obtained by additional condition or conditions given in question]

Here equation of circle be

$x^2+y^2-2x-4y+4=0$ ——— (i)

and the Equation of st. line

$x+2y-4=0$ ——— (ii)

Problem based on orthogonal circles :→

If two circles

$$x^2+y^2+2gx+2fy+c=0$$

and $x^2+y^2+2g_1x+2f_1y+c_1=0$

touches orthogonally

then we have

$$2gg_1+2ff_1=c+c_1$$

Problem 3 :→

Show that the circles

$$x^2+y^2-2ax+2by+c=0$$

and $x^2+y^2+2bx+2ay-c=0$

cut one another orthogonally

Here Given circles are

$$x^2+y^2-2ax+2by+c=0 \quad -(1)$$

$$x^2+y^2+2bx+2ay-c=0 \quad -(11)$$

these two circles intersect each other orthogonally.

Here $2g=-2a$, $2f=2b$, $c=c$ | $2g_1=2b \Rightarrow g_1=b$, $2f_1=2a \Rightarrow f_1=a$, $c_1=-c$

$$\Rightarrow 2gg_1+2ff_1=(c+c_1)=0$$

$$-2a\times b+2b\times a-(c-c)$$

$$=-ab+ab-0$$

$\Rightarrow 0$. Hence the result.